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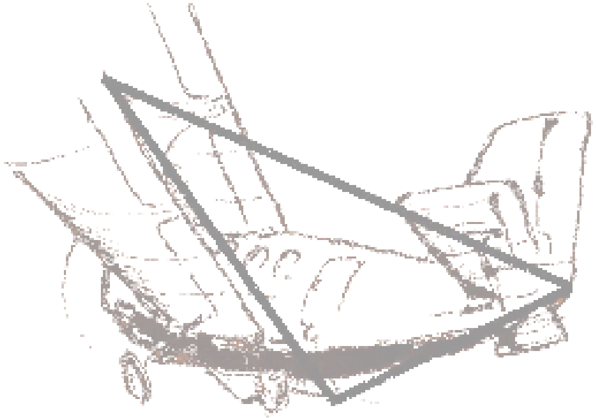
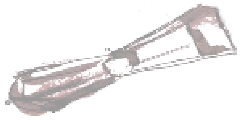


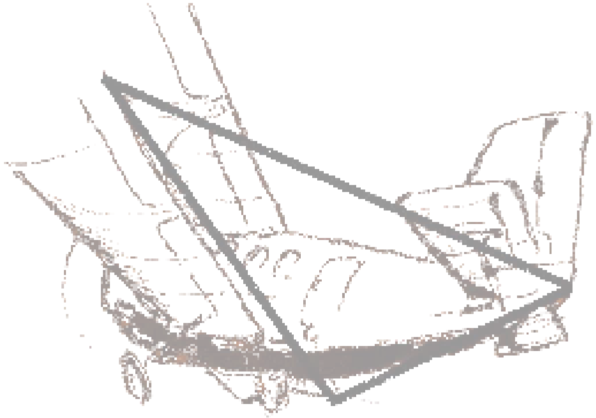
Iterated extended Kalman filter for airborne electromagnetic data inversion



Evgeny Karshakov

V.A. Trapeznikov Institute of Control Sciences of Russian Academy of Sciences
LLC Geotechnologies
Moscow, Russia

Pictures: A.K. Volkovitsky



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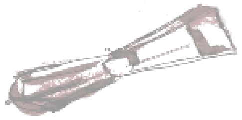
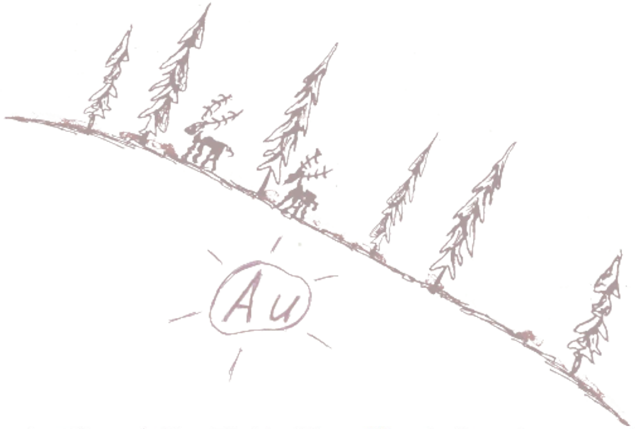


• AEM 1D invertion methods: a review....................7



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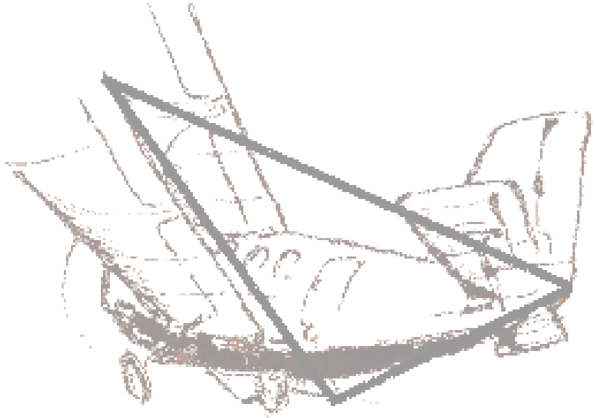

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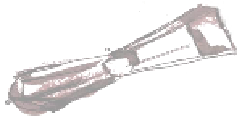

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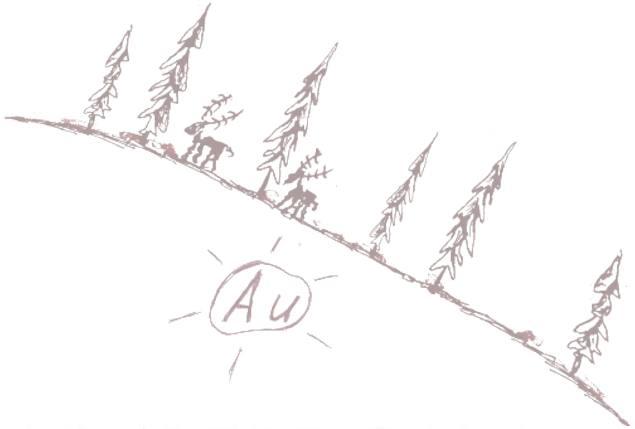

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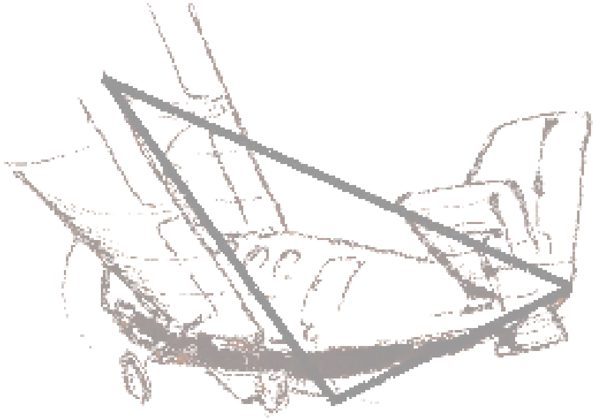

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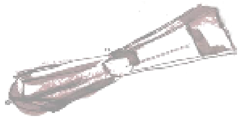

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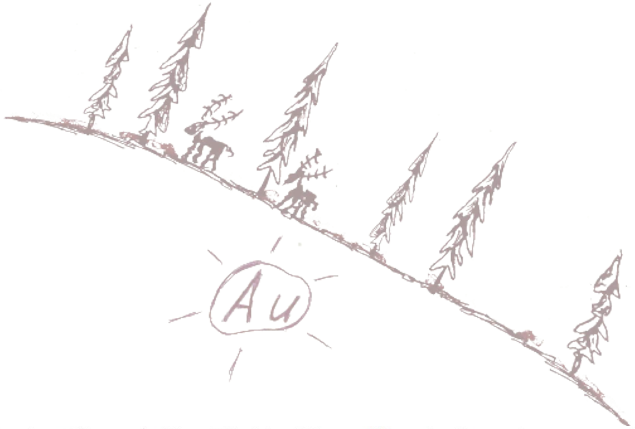

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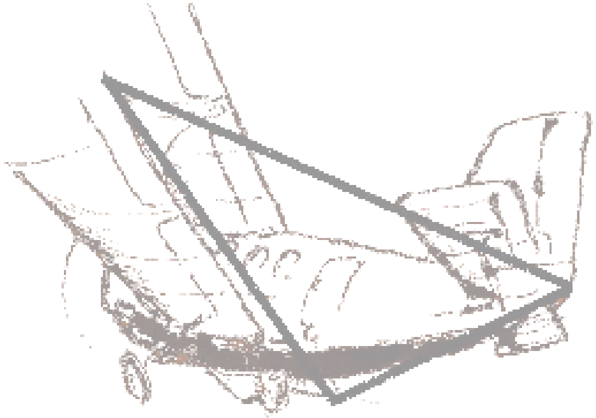

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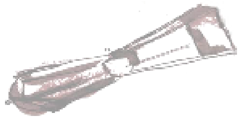

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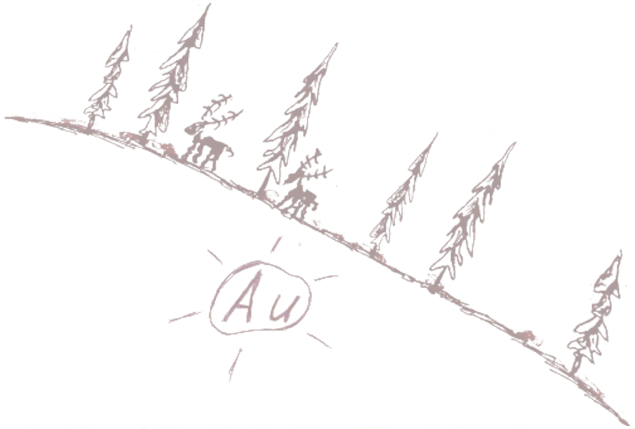

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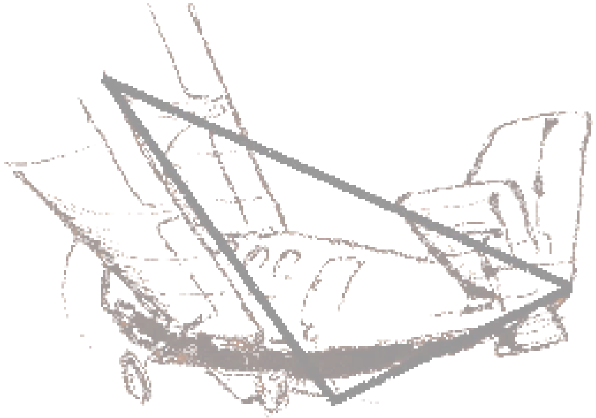

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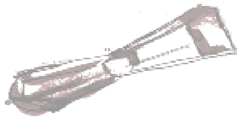

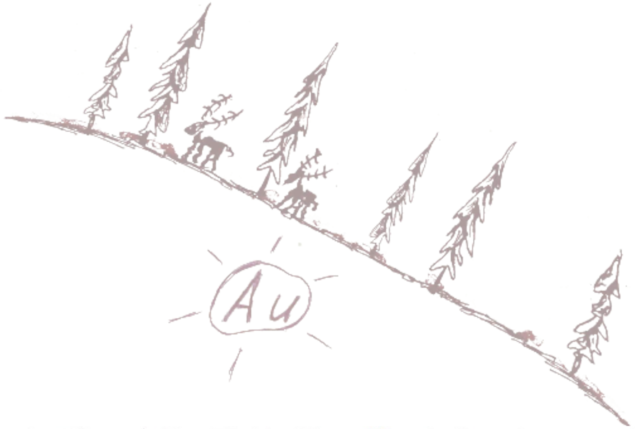

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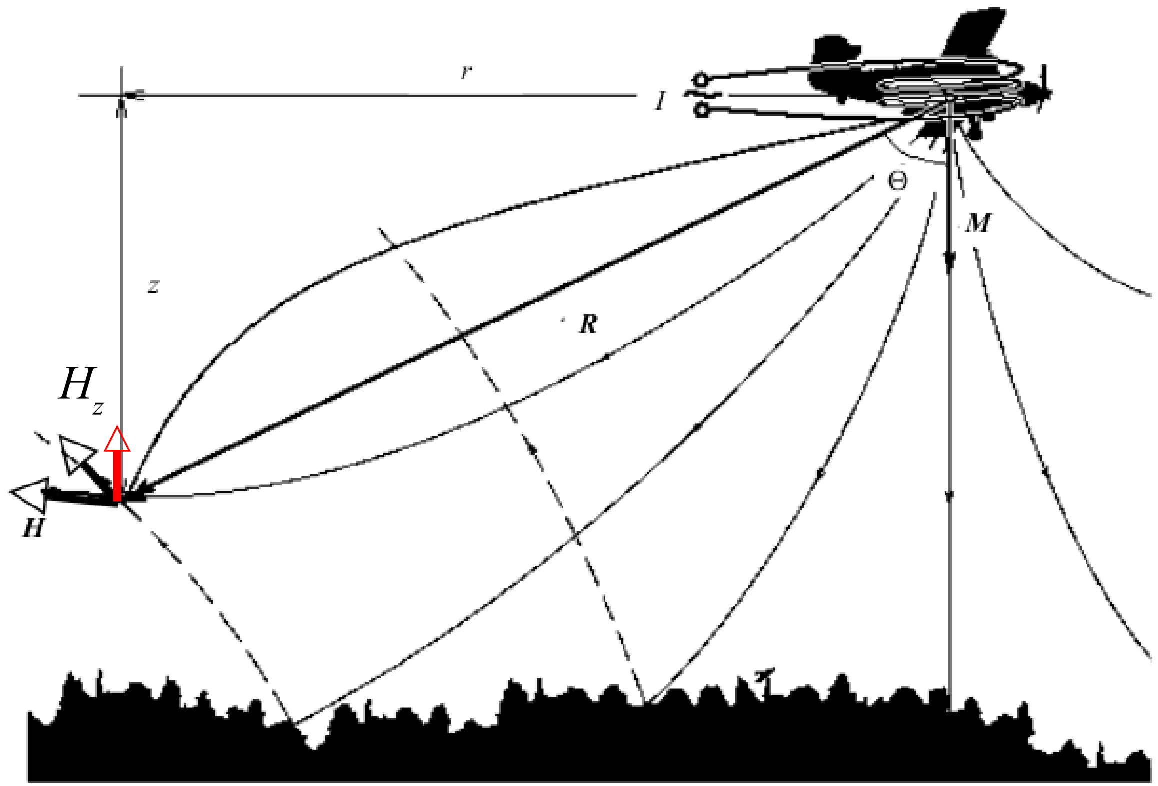
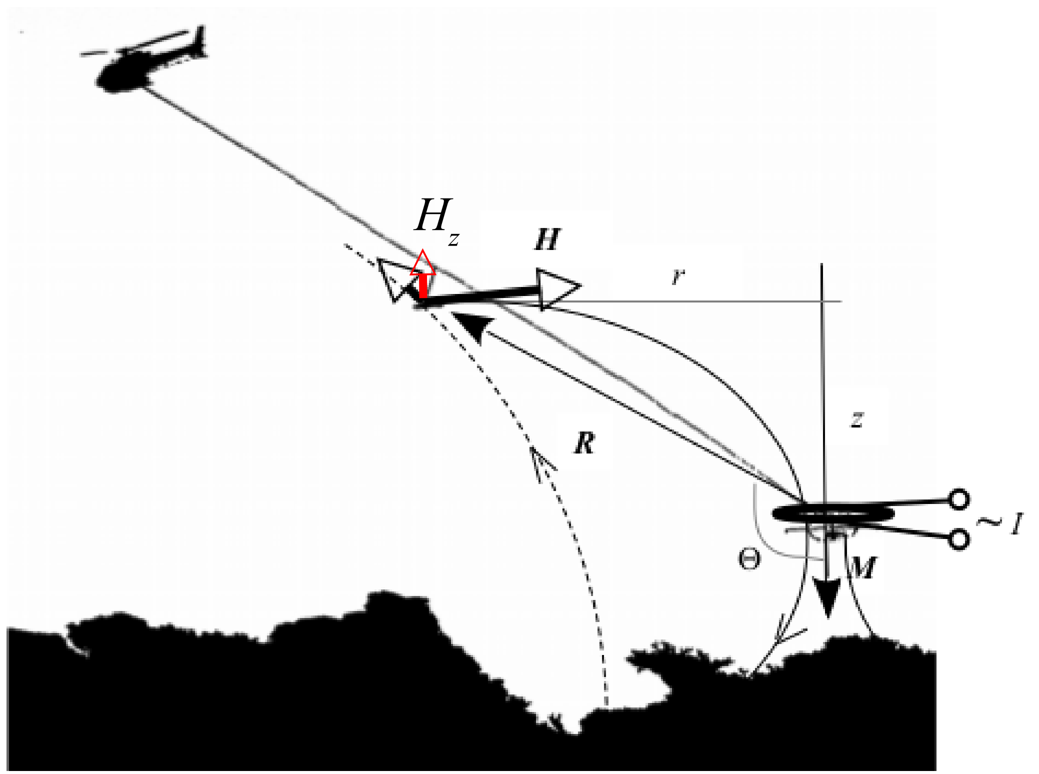
AEM 1D invertion methods: a review





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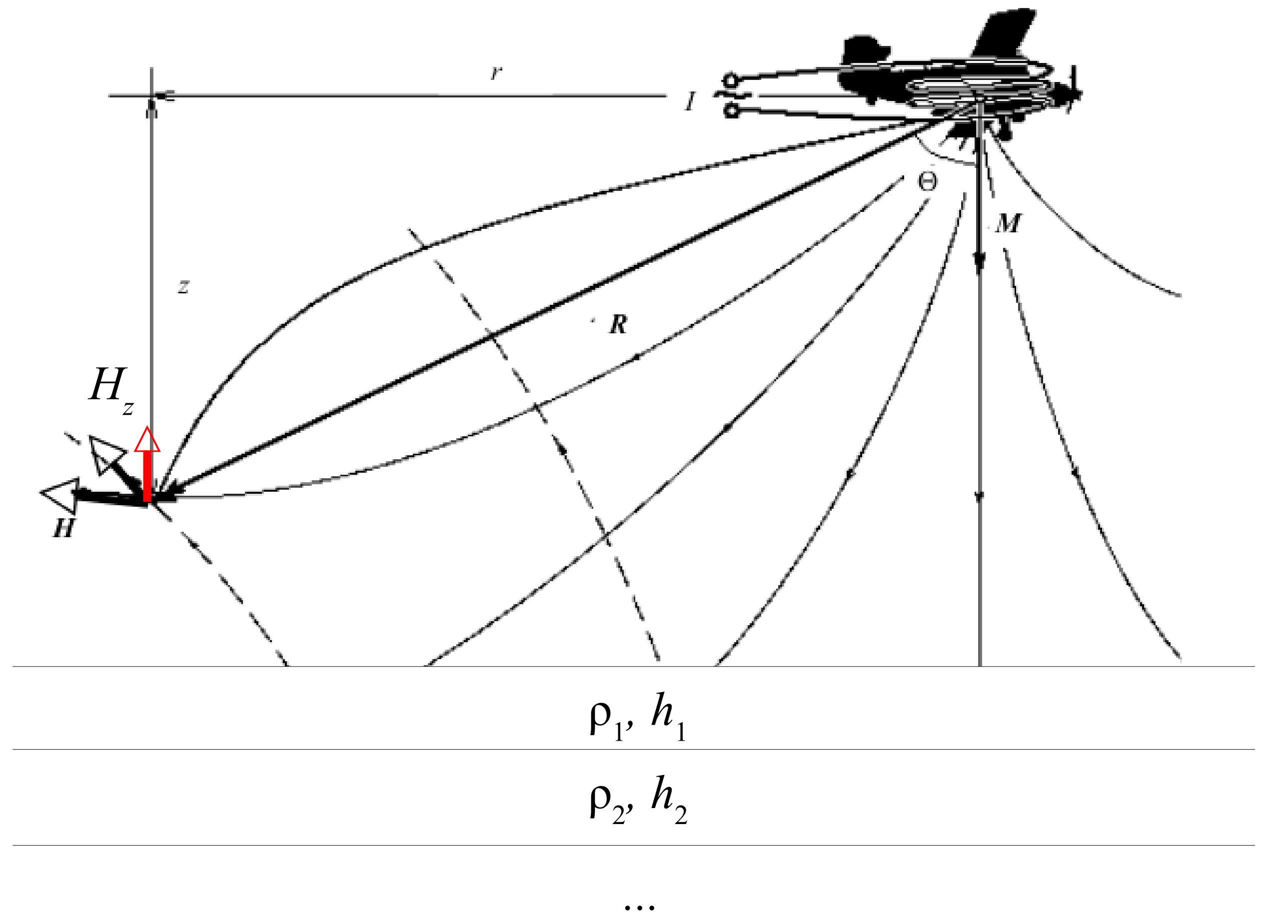

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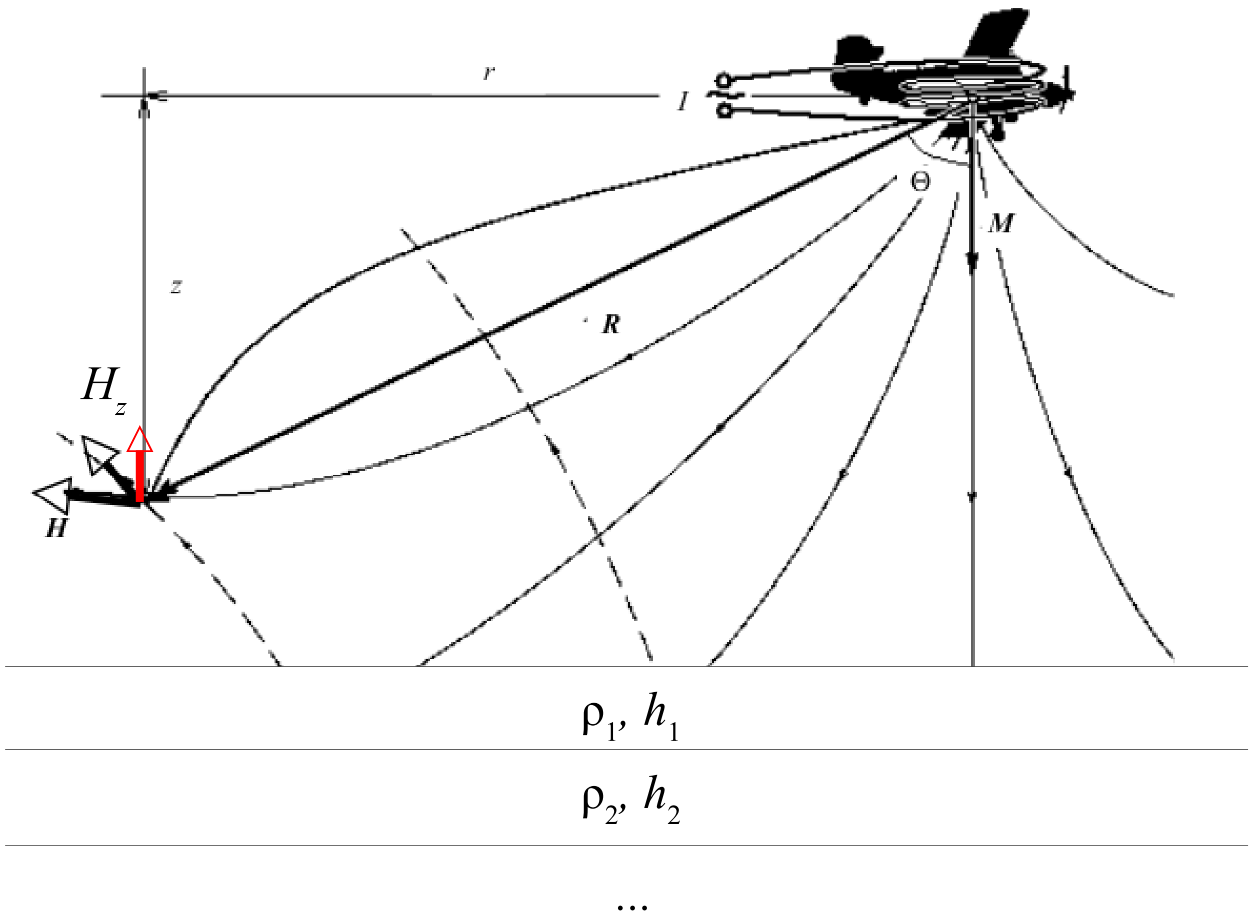


Guillemoteau, J., Sailhac, P. and Béhaegel, M., 2011, Regularization strategy for the layered inversion of airborne transient electromagnetic data: application to in-loop data acquired over the basin of Franceville (Gabon). Geophysical Prospecting, 59, 1132–1143

Chang-Chun, Y., Xiu-Yan, R., Yun-He, L., Yan-Fu, Q., Chang-Kai, Q. and Jing, C., 2015, Review on airborne electromagnetic inverse theory and applications. Geophysics, 80(4), W17–W31

Auken, E., Boesen, T. and Christiansen, A.V., 2017, A review of airborne electromagnetic methods with focus on geotechnical and hydrological applications from 2007 to 2017. Chapter 2 in: Advances in Geophysics, 58, 47–93

AEM 1D invertion methods: a review

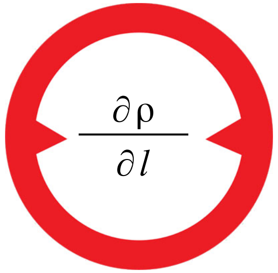
Laterally constrained LCI

$$\tilde{\mathbf{x}}^{+} = \tilde{\mathbf{x}}^{-} + \left[\mathbf{H}^{T}\mathbf{R}^{-1}\mathbf{H} + \mathbf{G}^{T}\mathbf{G}\right]^{-1}\mathbf{H}^{T}\mathbf{R}^{-1}\left(\mathbf{z} - \mathbf{H}\,\tilde{\mathbf{x}}^{-}\right)$$

$$\tilde{\mathbf{x}}_{j}^{-} = \tilde{\mathbf{x}}_{j-1}^{+}$$

$$\mathbf{G} = \lambda\,\mathbf{I}$$

$$\mathbf{I} = \begin{pmatrix} 1 & 0 & & \\ 0 & 1 & 0 & \\ & \cdots & & \\ & & \cdots & \\ & 0 & 1 & 0 \\ & & 0 & 1 \end{pmatrix}$$



AEM 1D invertion methods: a review

Laterally constrained LCI

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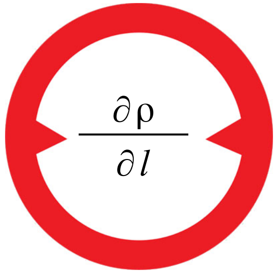
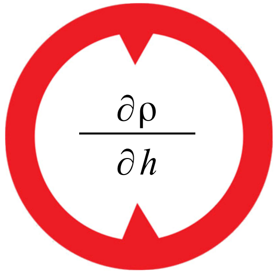


Vertically constrained VCI

$$\tilde{\mathbf{x}}^{+} = \tilde{\mathbf{x}}^{-} + \left[\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H} + \mathbf{D}^T \mathbf{D}\right]^{-1} \mathbf{H}^T \mathbf{R}^{-1} \left(\mathbf{z} - \mathbf{H}\,\tilde{\mathbf{x}}^{-}\right)$$

$$\tilde{\mathbf{x}}^{-} = 0$$

$$\mathbf{D} = \begin{pmatrix} 1/\delta h_1 & -1/\delta h_1 & & \\ 1/\delta h_2^2 & -2/\delta h_2^2 & 1/\delta h_2^2 & \\ & \cdots & & \\ & & \cdots & \\ & 1/\delta h_{N-1}^2 & -2/\delta h_{N-1}^2 & 1/\delta h_{N-1}^2 \\ & & -1/\delta h_N & 1/\delta h_N \end{pmatrix}$$



AEM 1D invertion methods: a review

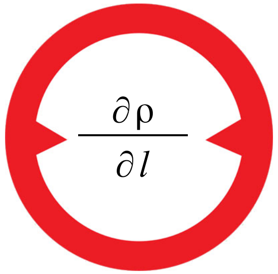
Laterally constrained LCI

$$\tilde{\mathbf{x}}^{+}=\tilde{\mathbf{x}}^{-}+[\mathbf{H}^{T}\mathbf{R}^{-1}\mathbf{H}+\mathbf{G}^{T}\mathbf{G}]^{-1}\mathbf{H}^{T}\mathbf{R}^{-1}(\mathbf{z}-\mathbf{H}\tilde{\mathbf{x}}^{-})$$

$$\tilde{\mathbf{x}}_{j}^{-}=\tilde{\mathbf{x}}_{j-1}^{+}$$

$$\mathbf{G}=\lambda\mathbf{I}$$

$$\mathbf{I}=\begin{pmatrix} 1 & 0 & & & \\ 0 & 1 & 0 & & \\ & \cdots & & & \\ & & \cdots & & \\ & 0 & 1 & 0 & \\ & & 0 & 1 & \end{pmatrix}$$

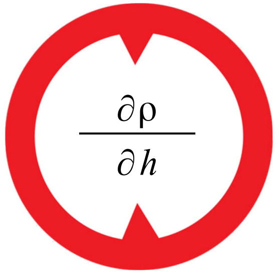


Vertically constrained VCI

$$\tilde{\mathbf{x}}^{+}=\tilde{\mathbf{x}}^{-}+[\mathbf{H}^{T}\mathbf{R}^{-1}\mathbf{H}+\mathbf{D}^{T}\mathbf{D}]^{-1}\mathbf{H}^{T}\mathbf{R}^{-1}(\mathbf{z}-\mathbf{H}\tilde{\mathbf{x}}^{-})$$

$$\tilde{\mathbf{x}}^{-}=0$$

$$\mathbf{D}=\begin{pmatrix} 1/\delta h_1 & -1/\delta h_1 & & \\ 1/\delta h_2^2 & -2/\delta h_2^2 & 1/\delta h_2^2 & \\ & \cdots & & \\ & & \cdots & \\ & 1/\delta h_{N-1}^2 & -2/\delta h_{N-1}^2 & 1/\delta h_{N-1}^2 \\ & & -1/\delta h_N & 1/\delta h_N \end{pmatrix}$$



Singular value decomposition SVD

$$\tilde{\mathbf{x}}^{+}=\tilde{\mathbf{x}}^{-}+[\mathbf{H}^{T}\mathbf{R}^{-1}\mathbf{H}]^{-1}\mathbf{H}^{T}\mathbf{R}^{-1}(\mathbf{z}-\mathbf{H}\tilde{\mathbf{x}}^{-})$$

$$\tilde{\mathbf{x}}^{-}=0$$

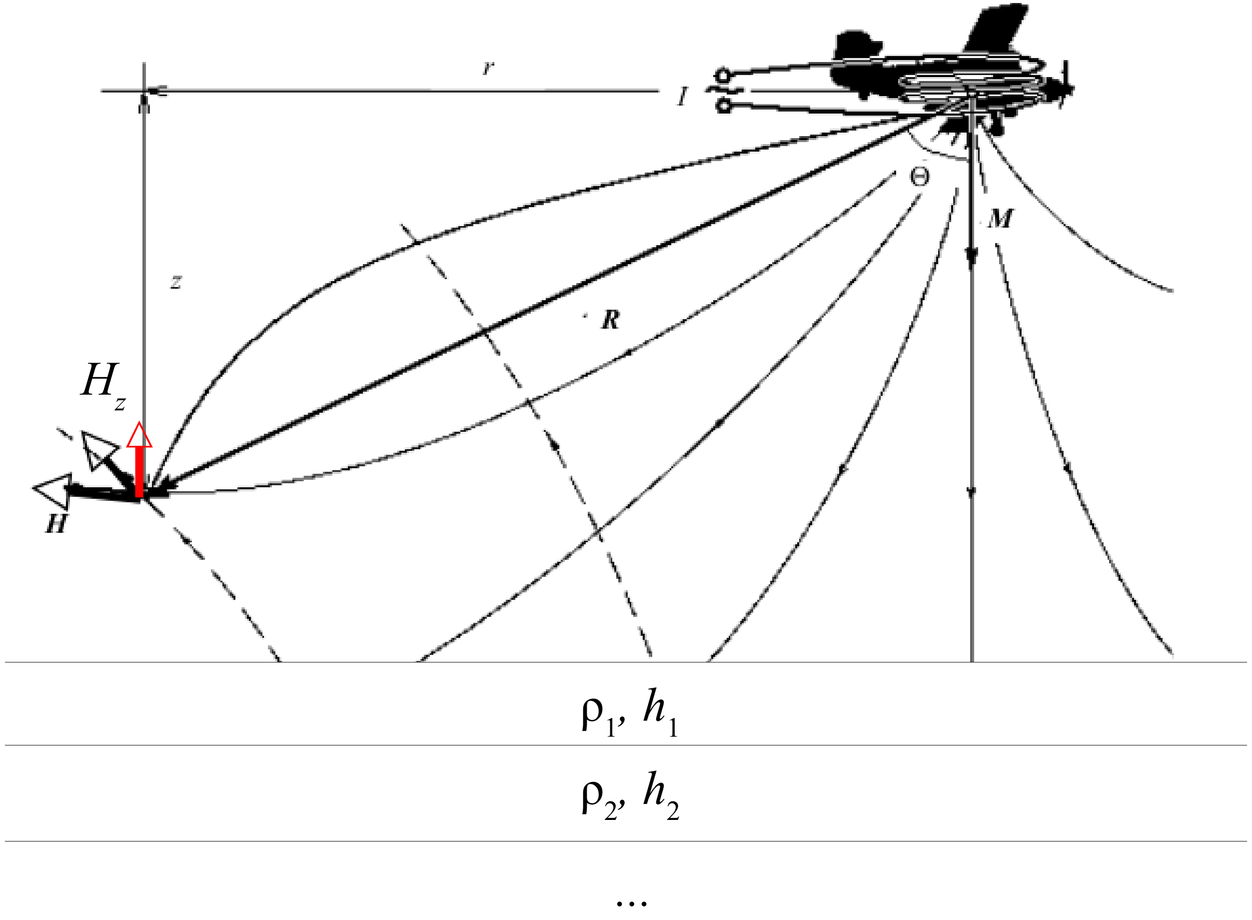
$$\mathbf{R}=\mathbf{I}$$

$$[\mathbf{H}^{T}\mathbf{R}^{-1}\mathbf{H}]^{-1}\mathbf{H}^{T}\mathbf{R}^{-1} \rightarrow \mathbf{V}\boldsymbol{\Lambda}^{-1}\mathbf{U}^{T}$$

A stochastic estimation problem

$$t_j: \quad \mathbf{z}_j = \left(\mathrm{Re}\, H_z(\omega_0), \mathrm{Im}\, H_z(\omega_0), \ldots, \mathrm{Re}\, H_z(\omega_K), \mathrm{Im}\, H_z(\omega_K), \frac{dH_z(\delta t_0)}{dt}, \ldots, d\frac{H_z(\delta t_S)}{dt} \right), \quad \mathbf{z}_j \in \mathbf{R}^N$$

$$\mathbf{x}_j = (\ln \rho_{1,} \ldots, \ln \rho_m, \ln h_1, \ldots, \ln h_{m-1}), \quad \mathbf{x}_j \in \mathbf{R}^M$$

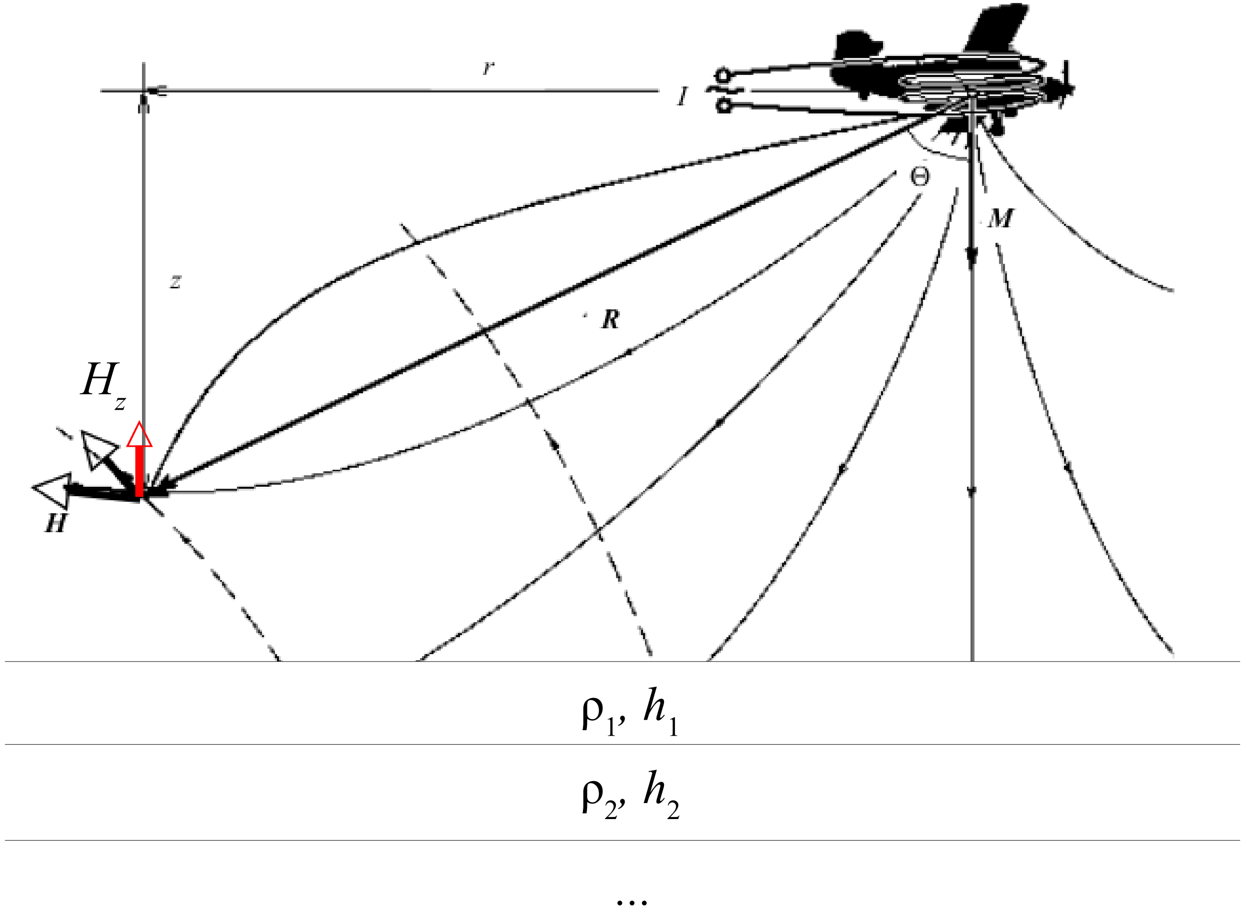


A stochastic estimation problem

$$t_j:\quad \mathbf{z}_j=\left(\mathrm{Re}\,H_z(\omega_0),\,\mathrm{Im}\,H_z(\omega_0),\ldots,\mathrm{Re}\,H_z(\omega_K),\,\mathrm{Im}\,H_z(\omega_K),\frac{dH_z(\delta t_0)}{dt},\ldots,d\frac{H_z(\delta t_S)}{dt}\right),\quad \mathbf{z}_j\in\mathbf{R}^N$$

$$\mathbf{x}_j=(\ln\rho_1,\ldots,\ln\rho_m,\ln h_1,\ldots,\ln h_{m-1}),\quad \mathbf{x}_j\in\mathbf{R}^M$$

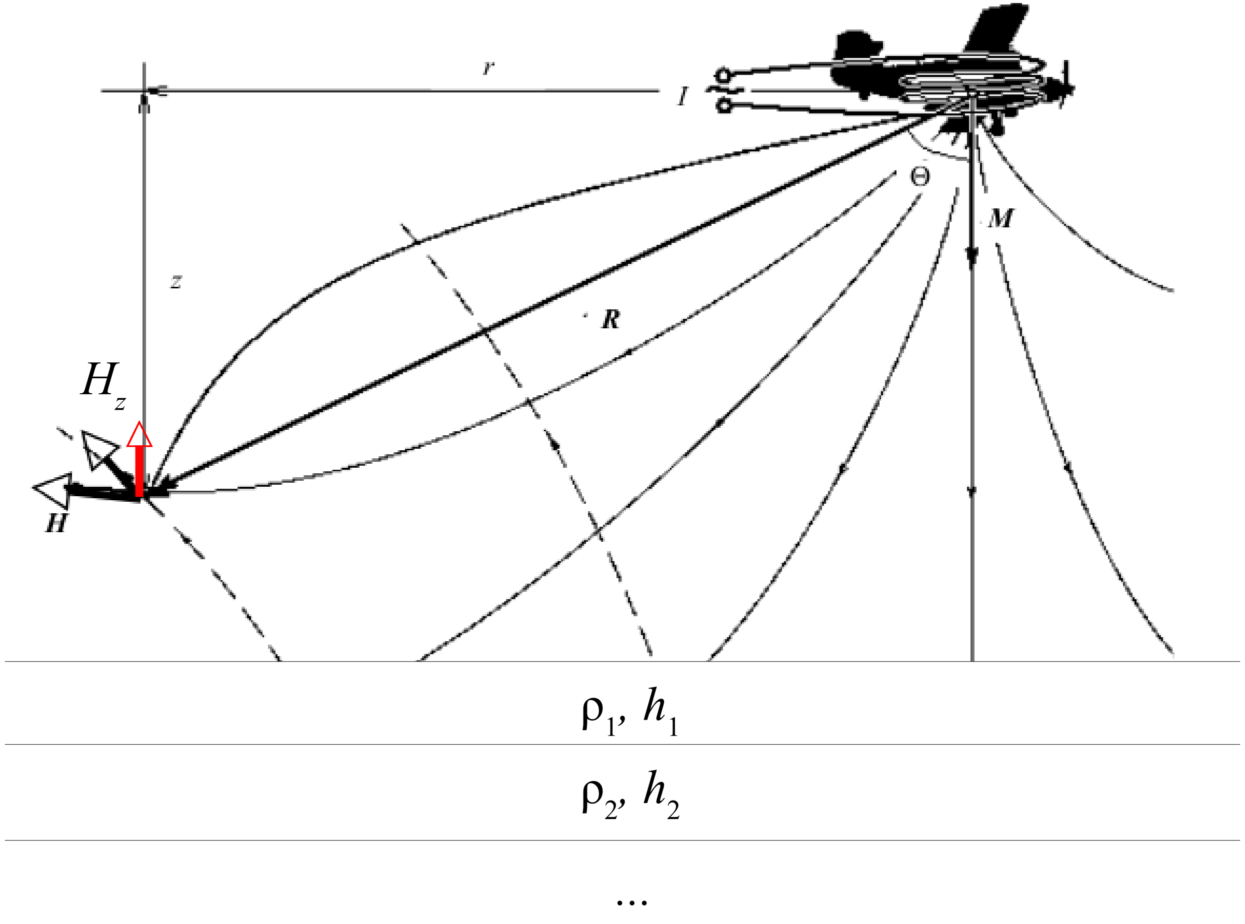
$$\mathbf{z}_j=\mathbf{h}_j(\mathbf{x}_j)+\mathbf{r}_j,\quad E[\mathbf{r}_j]=0,\quad E[\mathbf{r}_j\mathbf{r}_k^T]=\mathbf{R}_j\delta_{jk},$$



A stochastic estimation problem

$$t_j:\quad \mathbf{z}_j=\left(\operatorname{Re}H_z(\omega_0),\operatorname{Im}H_z(\omega_0),\dots,\operatorname{Re}H_z(\omega_K),\operatorname{Im}H_z(\omega_K),\frac{dH_z(\delta t_0)}{dt},\dots,d\frac{H_z(\delta t_S)}{dt}\right),\quad \mathbf{z}_j\in\mathbf{R}^N$$

$$\mathbf{x}_j=(\ln\rho_1,\dots,\ln\rho_m,\ln h_1,\dots,\ln h_{m-1}),\quad \mathbf{x}_j\in\mathbf{R}^M$$



$$\mathbf{z}_j=\mathbf{h}_j(\mathbf{x}_j)+\mathbf{r}_j,\quad E[\mathbf{r}_j]=0,\quad E[\mathbf{r}_j\mathbf{r}_k^T]=\mathbf{R}_j\delta_{jk},$$

Zhdanov, M.S., 2009, Geophysical Electromagnetic Theory and Methods: Elsevier

$$H_z(r,z,h_T,\omega)=-\frac{1}{2\pi}\int_0^\infty u(n_0,z,h_T,\omega)J_0(n_0 r)n_0^2\,dn_0,$$

$$u(n_0,z,h_T,\omega)=\frac{M e^{-n_0(z+h_T)}}{2}\cdot\frac{n_1-n_0R^*}{n_1+n_0R^*},$$

$$R^*=\operatorname{th}\left\{n_1h_1+\operatorname{arcth}\left[\frac{n_1}{n_2}\operatorname{th}\left(n_2h_2+\cdots\left(n_{K-1}h_{K-1}+\operatorname{arcth}\frac{n_{K-1}}{n_K}\right)\cdots\right)\right]\right\},$$

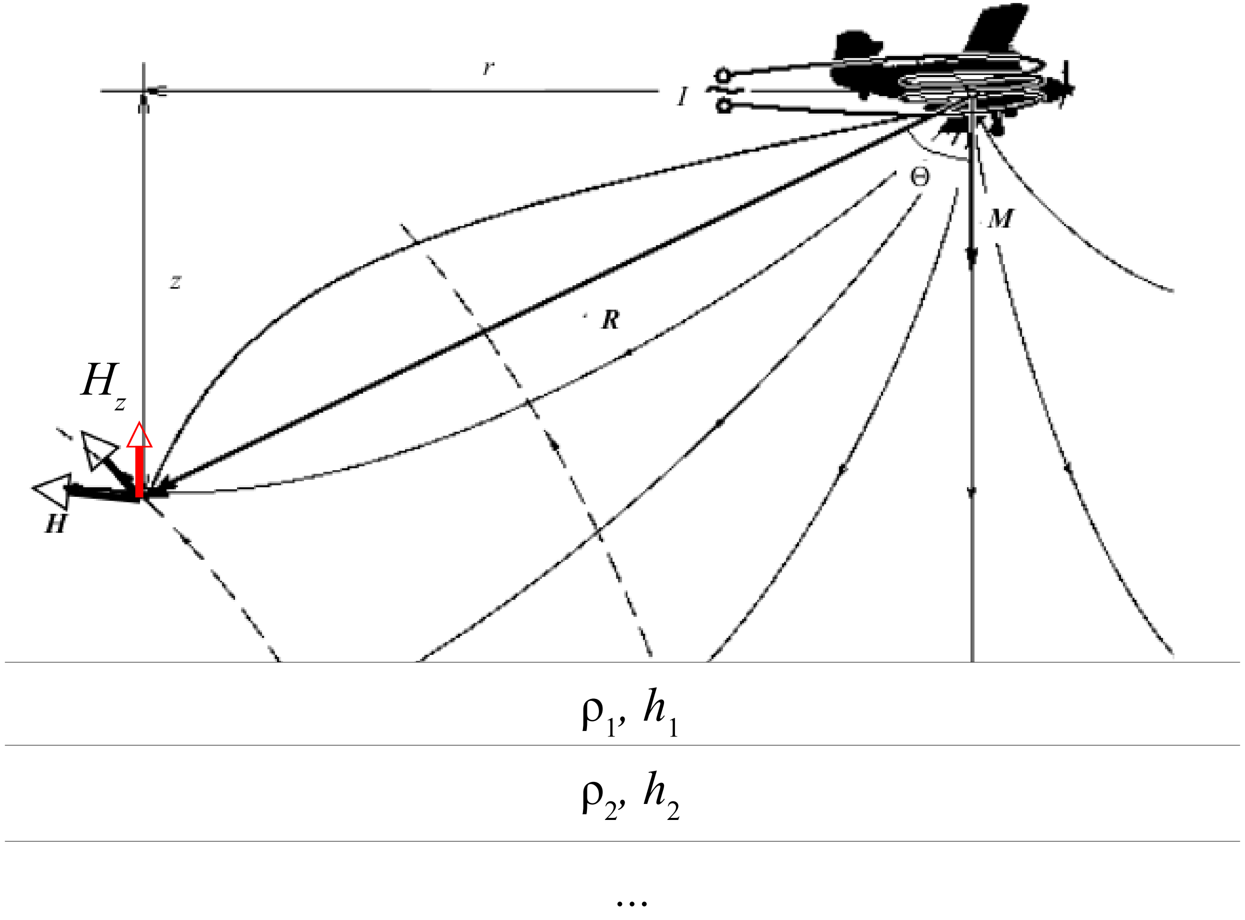
$$n_j=\sqrt{n_0^2-\frac{i\omega\mu_0}{\rho_j}},\quad \operatorname{Re}n_j>0,$$

$$H_z(t)=\frac{1}{2\pi}\sum_{k=0}^{L}SH_z([1+2k]\omega_0)\cdot ST([1+2k]\omega_0)\cdot SR([1+2k]\omega_0)\cdot e^{-i[1+2k]\omega_0 t}$$

A stochastic estimation problem

$$t_j:\quad \mathbf{z}_j = \left(\operatorname{Re} H_z(\omega_0), \operatorname{Im} H_z(\omega_0), \dots, \operatorname{Re} H_z(\omega_K), \operatorname{Im} H_z(\omega_K), \frac{dH_z(\delta t_0)}{dt}, \dots, d\frac{H_z(\delta t_S)}{dt}\right), \quad \mathbf{z}_j \in \mathbf{R}^N$$

$$\mathbf{x}_j = (\ln\rho_1, \dots, \ln\rho_m, \ln h_1, \dots, \ln h_{m-1}), \quad \mathbf{x}_j \in \mathbf{R}^M$$



$$\mathbf{z}_j = \mathbf{h}_j(\mathbf{x}_j) + \mathbf{r}_j, \quad E[\mathbf{r}_j] = 0, \quad E[\mathbf{r}_j \mathbf{r}_k^T] = \mathbf{R}_j \delta_{jk},$$

$$\mathbf{x}_{j+1} = \mathbf{f}_j(\mathbf{x}_j) + \mathbf{q}_j, \quad E[\mathbf{q}_j] = 0, \quad E[\mathbf{q}_j \mathbf{q}_k^T] = \mathbf{Q}_j \delta_{jk}$$

$$\tilde{\mathbf{x}}_0^- = E[\mathbf{x}_0], \quad \mathbf{P}_0^- = E[\Delta\mathbf{x}_0 \Delta\mathbf{x}_0^T].$$

A stochastic estimation problem

$$t_j: \quad \mathbf{z}_j = \left(\mathrm{Re}\, H_z(\omega_0), \mathrm{Im}\, H_z(\omega_0), \dots, \mathrm{Re}\, H_z(\omega_K), \mathrm{Im}\, H_z(\omega_K), \frac{dH_z(\delta t_0)}{dt}, \dots, d\frac{H_z(\delta t_S)}{dt} \right), \quad \mathbf{z}_j \in \mathbf{R}^N$$

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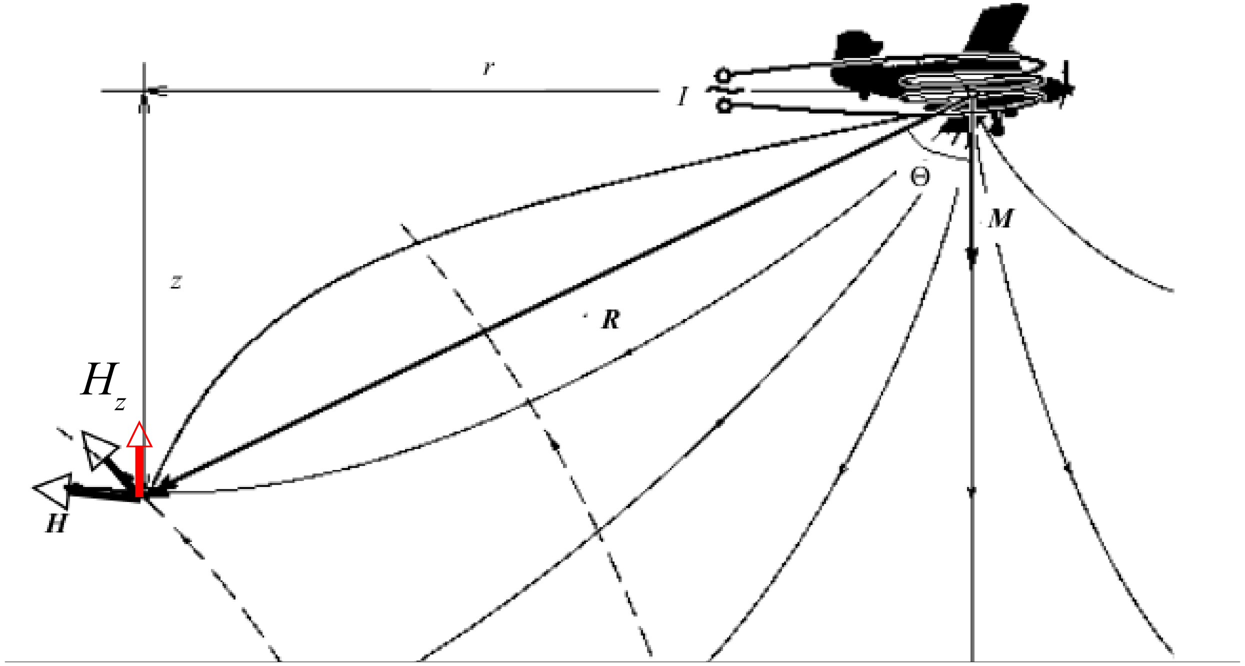

$\rho_1 \pm \delta\rho_1,\ h_1 \pm \delta h_1$

$\rho_2 \pm \delta\rho_2,\ h_2 \pm \delta h_2$

...

$$\mathbf{z}_j = \mathbf{h}_j(\mathbf{x}_j) + \mathbf{r}_j, \quad E[\mathbf{r}_j] = 0, \quad E[\mathbf{r}_j \mathbf{r}_k^T] = \mathbf{R}_j \delta_{jk},$$

$$\mathbf{x}_{j+1} = \mathbf{x}_j + \mathbf{q}_j, \quad E[\mathbf{q}_j] = 0, \quad E[\mathbf{q}_j \mathbf{q}_k^T] = V^2 \mathbf{Q}_j \delta_{jk}$$

$$\tilde{\mathbf{x}}_0^- = E[\mathbf{x}_0], \quad \mathbf{P}_0^- = E[\Delta \mathbf{x}_0 \Delta \mathbf{x}_0^T].$$

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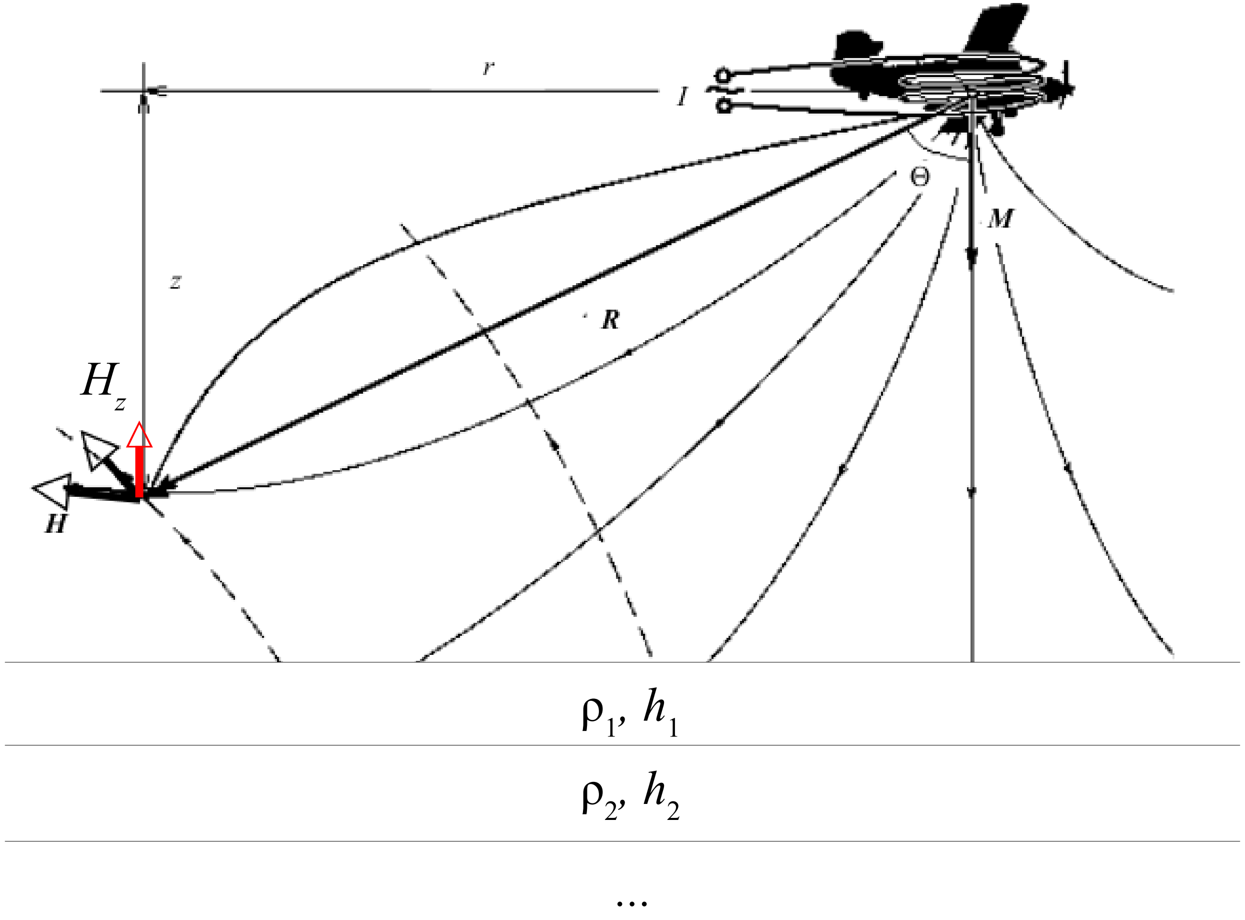
Extended Kalman Filter

$$t_j: \quad \mathbf{z}_j = \left(\operatorname{Re} H_z(\omega_0), \operatorname{Im} H_z(\omega_0), \ldots, \operatorname{Re} H_z(\omega_K), \operatorname{Im} H_z(\omega_K), \frac{dH_z(\delta t_0)}{dt}, \ldots, d\frac{H_z(\delta t_S)}{dt} \right), \quad \mathbf{z}_j \in \mathbf{R}^N$$

$$\mathbf{x}_j = (\ln \rho_1, \ldots, \ln \rho_m, \ln h_1, \ldots, \ln h_{m-1}), \quad \mathbf{x}_j \in \mathbf{R}^M$$



$$\mathbf{z}_j = \mathbf{h}_j(\mathbf{x}_j) + \mathbf{r}_j, \quad E[\mathbf{r}_j] = 0, \quad E[\mathbf{r}_j \mathbf{r}_k^T] = \mathbf{R}_j \delta_{jk},$$

$$\mathbf{x}_{j+1} = \mathbf{f}_j(\mathbf{x}_j) + \mathbf{q}_j, \quad E[\mathbf{q}_j] = 0, \quad E[\mathbf{q}_j \mathbf{q}_k^T] = \mathbf{Q}_j \delta_{jk}$$

$$\tilde{\mathbf{x}}_0^- = E[\mathbf{x}_0], \quad \mathbf{P}_0^- = E[\Delta \mathbf{x}_0 \Delta \mathbf{x}_0^T].$$

Simon, D., 2006, Optimal State Estimation. Kalman, H∞ and Nonlinear Approaches: John Wiley & Sons, Inc., Hoboken, New Jersey.

Kalman, R., 1960, A new approach to linear filtering and prediction problems: ASME Journal of Basic Engineering, 82, 35-45.

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Extended Kalman Filter

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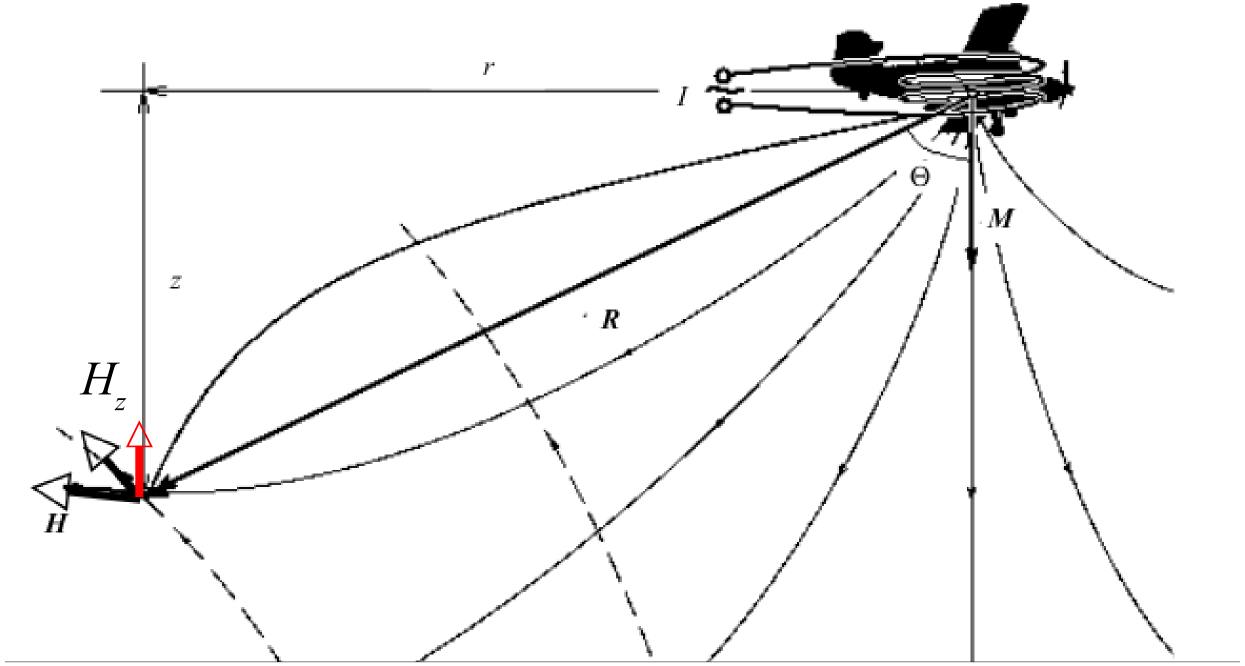

$\rho_1 \pm \delta\rho_1,\ h_1 \pm \delta h_1$

$\rho_2 \pm \delta\rho_2,\ h_2 \pm \delta h_2$

…

$$\mathbf{z}_j = \mathbf{h}_j(\mathbf{x}_j) + \mathbf{r}_j, \quad E[\mathbf{r}_j] = 0, \quad E[\mathbf{r}_j \mathbf{r}_k^T] = \mathbf{R}_j \delta_{jk},$$

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Simon, D., 2006, Optimal State Estimation. Kalman, H∞ and Nonlinear Approaches: John Wiley & Sons, Inc., Hoboken, New Jersey.

1. Prognosis step

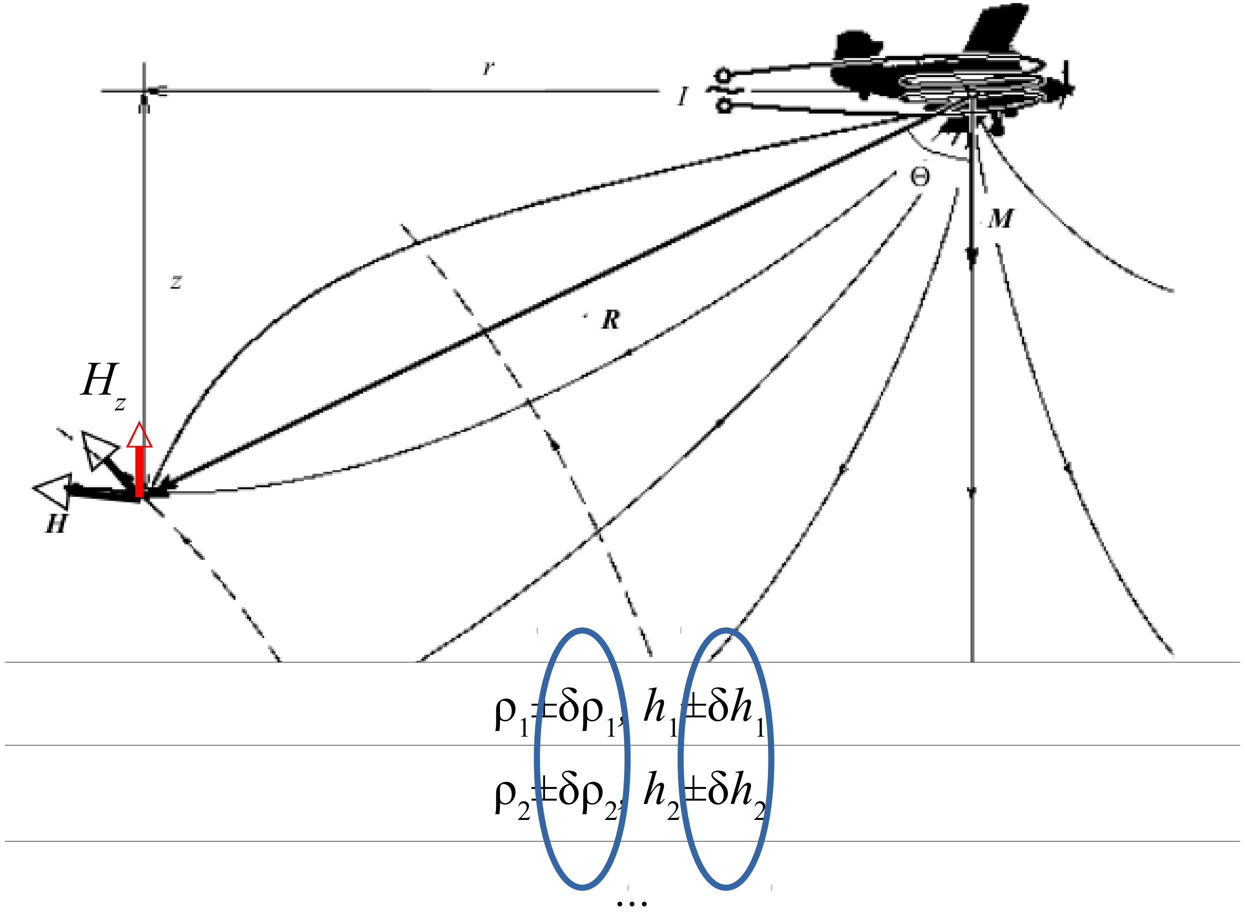
$$\tilde{\mathbf{x}}_j^- = \mathbf{f}_{j-1}(\tilde{\mathbf{x}}_{j-1}^+),$$

$$\mathbf{P}_j^- = \mathbf{A}_{j-1} \mathbf{P}_{j-1}^+ \mathbf{A}_{j-1}^T + \mathbf{Q}_{j-1}, \quad \mathbf{A}_{j-1} = \frac{\partial \mathbf{f}_{j-1}}{\partial \mathbf{x}}.$$

Extended Kalman Filter

$$t_j:\quad \mathbf{z}_j=\left(\operatorname{Re}H_z(\omega_0),\operatorname{Im}H_z(\omega_0),\dots,\operatorname{Re}H_z(\omega_K),\operatorname{Im}H_z(\omega_K),\frac{dH_z(\delta t_0)}{dt},\dots,d\frac{H_z(\delta t_S)}{dt}\right),\quad \mathbf{z}_j\in\mathbf{R}^N$$

$$\mathbf{x}_j=(\ln\rho_{1,},\dots,\ln\rho_m,\ln h_1,\dots,\ln h_{m-1}),\quad \mathbf{x}_j\in\mathbf{R}^M$$



$$\mathbf{z}_j=\mathbf{h}_j(\mathbf{x}_j)+\mathbf{r}_j,\quad E[\mathbf{r}_j]=0,\quad E[\mathbf{r}_j\mathbf{r}_k^T]=\mathbf{R}_j\delta_{jk},$$

$$\mathbf{x}_{j+1}=\mathbf{f}_j(\mathbf{x}_j)+\mathbf{q}_j,\quad E[\mathbf{q}_j]=0,\quad E[\mathbf{q}_j\mathbf{q}_k^T]=\mathbf{Q}_j\delta_{jk}$$

$$\tilde{\mathbf{x}}_0^-=E[\mathbf{x}_0],\quad \mathbf{P}_0^-=E[\Delta\mathbf{x}_0\Delta\mathbf{x}_0^T].$$

Simon, D., 2006, Optimal State Estimation. Kalman, H∞ and Nonlinear Approaches: John Wiley & Sons, Inc., Hoboken, New Jersey.

2. Correction step

$$\tilde{\mathbf{x}}_j^{k+}=\tilde{\mathbf{x}}_j^{k-}+\mathbf{K}_j^k(\mathbf{z}_j-\mathbf{h}_j(\tilde{\mathbf{x}}_j^{k-})),$$

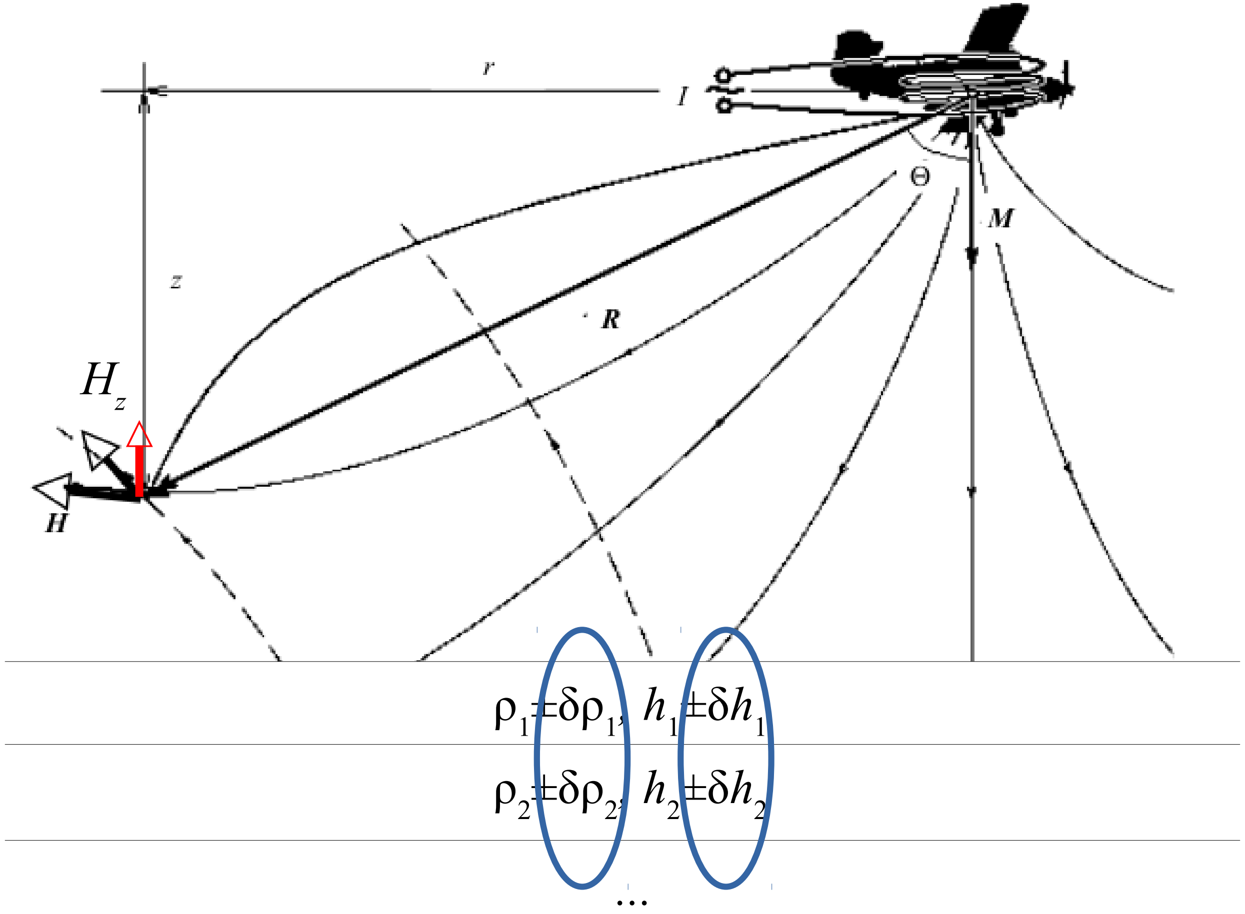
$$\mathbf{P}_j^{k+}=\left(\mathbf{I}-\mathbf{K}_j^k\frac{\partial\mathbf{h}_j(\tilde{\mathbf{x}}_j^k)}{\partial\mathbf{x}}\right)\mathbf{P}_j^{k-},$$

$$\mathbf{K}_j^k=\mathbf{P}_j^{k-}\left(\frac{\partial\mathbf{h}_j(\tilde{\mathbf{x}}_j^{k-})}{\partial\mathbf{x}}\right)^T\left[\frac{\partial\mathbf{h}_j(\tilde{\mathbf{x}}_j^{k-})}{\partial\mathbf{x}}\mathbf{P}_j^{k-}\left(\frac{\partial\mathbf{h}_j(\tilde{\mathbf{x}}_j^{k-})}{\partial\mathbf{x}}\right)^T+\mathbf{R}_j\right]^{-1}.$$

Iterated Extended Kalman Filter

$$t_j:\quad \mathbf{z}_j = \left(\mathrm{Re}\,H_z(\omega_0), \mathrm{Im}\,H_z(\omega_0), \dots, \mathrm{Re}\,H_z(\omega_K), \mathrm{Im}\,H_z(\omega_K), \frac{dH_z(\delta t_0)}{dt}, \dots, d\frac{H_z(\delta t_S)}{dt}\right), \quad \mathbf{z}_j \in \mathbf{R}^N$$

$$\mathbf{x}_j = (\ln\rho_1, \dots, \ln\rho_m, \ln h_1, \dots, \ln h_{m-1}), \quad \mathbf{x}_j \in \mathbf{R}^M$$



$$\mathbf{z}_j = \mathbf{h}_j(\mathbf{x}_j) + \mathbf{r}_j, \quad E[\mathbf{r}_j] = 0, \quad E[\mathbf{r}_j \mathbf{r}_k^T] = \mathbf{R}_j \delta_{jk},$$

$$\mathbf{x}_{j+1} = \mathbf{f}_j(\mathbf{x}_j) + \mathbf{q}_j, \quad E[\mathbf{q}_j] = 0, \quad E[\mathbf{q}_j \mathbf{q}_k^T] = \mathbf{Q}_j \delta_{jk}$$

$$\tilde{\mathbf{x}}_0^- = E[\mathbf{x}_0], \quad \mathbf{P}_0^- = E[\Delta\mathbf{x}_0 \Delta\mathbf{x}_0^T].$$

Havlik, J. and Straka, O., 2015, Performance evaluation of iterated extended Kalman filter with variable step-length: Journal of Physics: Conference Series, 659, 012-022.

2. Iterated correction step (by k)

$$\tilde{\mathbf{x}}_j^{k-} = \tilde{\mathbf{x}}_j^{k-1+}, \quad \mathbf{P}_j^{k-} = \frac{\|\mathbf{z}_j - \mathbf{h}_j(\tilde{\mathbf{x}}_j^{k-1+})\|^2}{\|\mathbf{z}_j - \mathbf{h}_j(\tilde{\mathbf{x}}_j^{k-1-})\|^2} \mathbf{P}_j^{k-1-}.$$

$$\|\mathbf{z}_j - \mathbf{h}_j(\tilde{\mathbf{x}}_j^{k+})\| = \sqrt{(\mathbf{z}_j - \mathbf{h}_j(\tilde{\mathbf{x}}_j^{k+}))^T \mathbf{R}^{-1} (\mathbf{z}_j - \mathbf{h}_j(\tilde{\mathbf{x}}_j^{k+}))}.$$

Iterated Extended Kalman Filter

Laterally constrained LCI

$$\tilde{\mathbf{x}}^{+}=\tilde{\mathbf{x}}^{-}+\left[\mathbf{H}^{T}\mathbf{R}^{-1}\mathbf{H}+\mathbf{G}^{T}\mathbf{G}\right]^{-1}\mathbf{H}^{T}\mathbf{R}^{-1}\left(\mathbf{z}-\mathbf{H}\tilde{\mathbf{x}}^{-}\right)$$

$$\tilde{\mathbf{x}}_{j}^{-}=\tilde{\mathbf{x}}_{j-1}^{+}$$

$$\mathbf{G}=\lambda\mathbf{I}$$

$$\mathbf{I}=\begin{pmatrix} 1 & 0 & & & \\ 0 & 1 & 0 & & \\ & \cdots & & & \\ & & \cdots & & \\ & 0 & 1 & 0 & \\ & & 0 & 1 & \end{pmatrix}$$

Vertically constrained VCI

$$\tilde{\mathbf{x}}^{+}=\tilde{\mathbf{x}}^{-}+\left[\mathbf{H}^{T}\mathbf{R}^{-1}\mathbf{H}+\mathbf{D}^{T}\mathbf{D}\right]^{-1}\mathbf{H}^{T}\mathbf{R}^{-1}\left(\mathbf{z}-\mathbf{H}\tilde{\mathbf{x}}^{-}\right)$$

$$\tilde{\mathbf{x}}^{-}=0$$

$$\mathbf{D}=\begin{pmatrix} 1/\delta h_1 & -1/\delta h_1 & & \\ 1/\delta h_2^2 & -2/\delta h_2^2 & 1/\delta h_2^2 & \\ & \cdots & & \\ & & \cdots & \\ & 1/\delta h_{N-1}^2 & -2/\delta h_{N-1}^2 & 1/\delta h_{N-1}^2 \\ & & -1/\delta h_N & 1/\delta h_N \end{pmatrix}$$

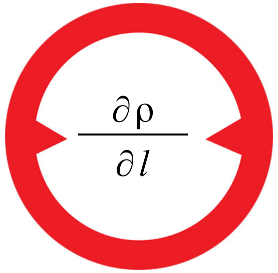
Singular value decomposition SVD

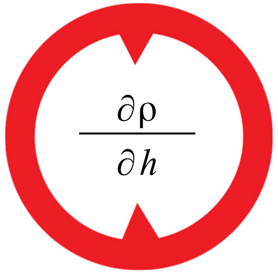
$$\tilde{\mathbf{x}}^{+}=\tilde{\mathbf{x}}^{-}+\left[\mathbf{H}^{T}\mathbf{R}^{-1}\mathbf{H}\right]^{-1}\mathbf{H}^{T}\mathbf{R}^{-1}\left(\mathbf{z}-\mathbf{H}\tilde{\mathbf{x}}^{-}\right)$$

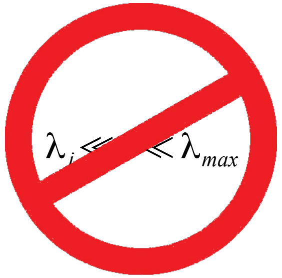
$$\tilde{\mathbf{x}}^{-}=0$$

$$\mathbf{R}=\mathbf{I}$$

$$\left[\mathbf{H}^{T}\mathbf{R}^{-1}\mathbf{H}\right]^{-1}\mathbf{H}^{T}\mathbf{R}^{-1} \rightarrow \mathbf{V}\mathbf{\Lambda}^{-1}\mathbf{U}^{T}$$







Iterated Extended Kalman Filter

Laterally constrained
LCI

$$\tilde{\mathbf{x}}^{+} = \tilde{\mathbf{x}}^{-} + [\mathbf{H}^{T}\mathbf{R}^{-1}\mathbf{H} + \mathbf{G}^{T}\mathbf{G}]^{-1}\mathbf{H}^{T}\mathbf{R}^{-1}(\mathbf{z} - \mathbf{H}\tilde{\mathbf{x}}^{-})$$

Vertically constrained
VCI

$$\tilde{\mathbf{x}}^{+} = \tilde{\mathbf{x}}^{-} + [\mathbf{H}^{T}\mathbf{R}^{-1}\mathbf{H} + \mathbf{D}^{T}\mathbf{D}]^{-1}\mathbf{H}^{T}\mathbf{R}^{-1}(\mathbf{z} - \mathbf{H}\tilde{\mathbf{x}}^{-})$$

Singular value decomposition
SVD

$$\tilde{\mathbf{x}}^{+} = \tilde{\mathbf{x}}^{-} + [\mathbf{H}^{T}\mathbf{R}^{-1}\mathbf{H}]^{-1}\mathbf{H}^{T}\mathbf{R}^{-1}(\mathbf{z} - \mathbf{H}\tilde{\mathbf{x}}^{-})$$

IEKF

$$\tilde{\mathbf{x}}^{+} = \tilde{\mathbf{x}}^{-} + \mathbf{P}^{-}\mathbf{H}^{T}[\mathbf{R} + \mathbf{H}\mathbf{P}^{-}\mathbf{H}^{T}]^{-1}(\mathbf{z} - \mathbf{H}\tilde{\mathbf{x}}^{-})$$

Iterated Extended Kalman Filter

Laterally constrained
LCI

$$\tilde{\mathbf{x}}^{+} = \tilde{\mathbf{x}}^{-} + [\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H} + \mathbf{G}^T \mathbf{G}]^{-1} \mathbf{H}^T \mathbf{R}^{-1} (\mathbf{z} - \mathbf{H}\tilde{\mathbf{x}}^{-})$$

Vertically constrained
VCI

$$\tilde{\mathbf{x}}^{+} = \tilde{\mathbf{x}}^{-} + [\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H} + \mathbf{D}^T \mathbf{D}]^{-1} \mathbf{H}^T \mathbf{R}^{-1} (\mathbf{z} - \mathbf{H}\tilde{\mathbf{x}}^{-})$$

Singular value decomposition
SVD

$$\tilde{\mathbf{x}}^{+} = \tilde{\mathbf{x}}^{-} + [\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H}]^{-1} \mathbf{H}^T \mathbf{R}^{-1} (\mathbf{z} - \mathbf{H}\tilde{\mathbf{x}}^{-})$$

IEKF

$$\tilde{\mathbf{x}}^{+} = \tilde{\mathbf{x}}^{-} + \mathbf{P}^{-} \mathbf{H}^T [\mathbf{R} + \mathbf{H}\mathbf{P}^{-} \mathbf{H}^T]^{-1} (\mathbf{z} - \mathbf{H}\tilde{\mathbf{x}}^{-})$$

The solution is the same!
According to the matrix inversion lemma.

$\mathbf{P}^{-1} = \mathbf{G}^T\mathbf{G}$, or $\mathbf{P}^{-1} = \mathbf{D}^T\mathbf{D}$, or $\mathbf{P}^{-1} = \mathbf{0}$

Simon, D., 2006, Optimal State Estimation. Kalman, H∞ and Nonlinear Approaches: John Wiley & Sons, Inc., Hoboken, New Jersey.

Iterated Extended Kalman Filter

Laterally constrained
LCI

$$\tilde{\mathbf{x}}^{+}=\tilde{\mathbf{x}}^{-}+\left[\mathbf{H}^{T}\mathbf{R}^{-1}\mathbf{H}+\mathbf{G}^{T}\mathbf{G}\right]^{-1}\mathbf{H}^{T}\mathbf{R}^{-1}\left(\mathbf{z}-\mathbf{H}\,\tilde{\mathbf{x}}^{-}\right)$$

Vertically constrained
VCI

$$\tilde{\mathbf{x}}^{+}=\tilde{\mathbf{x}}^{-}+\left[\mathbf{H}^{T}\mathbf{R}^{-1}\mathbf{H}+\mathbf{D}^{T}\mathbf{D}\right]^{-1}\mathbf{H}^{T}\mathbf{R}^{-1}\left(\mathbf{z}-\mathbf{H}\,\tilde{\mathbf{x}}^{-}\right)$$

Singular value decomposition
SVD

$$\tilde{\mathbf{x}}^{+}=\tilde{\mathbf{x}}^{-}+\left[\mathbf{H}^{T}\mathbf{R}^{-1}\mathbf{H}\right]^{-1}\mathbf{H}^{T}\mathbf{R}^{-1}\left(\mathbf{z}-\mathbf{H}\,\tilde{\mathbf{x}}^{-}\right)$$

IEKF

$$\tilde{\mathbf{x}}^{+}=\tilde{\mathbf{x}}^{-}+\mathbf{P}^{-}\mathbf{H}^{T}\left[\mathbf{R}+\mathbf{H}\,\mathbf{P}^{-}\mathbf{H}^{T}\right]^{-1}\left(\mathbf{z}-\mathbf{H}\,\tilde{\mathbf{x}}^{-}\right)$$

For an efficient numerical solution
Cholesky factorization: $\mathbf{P}=\mathbf{S}^{T}\mathbf{S}$
or LDL decomposition: $\mathbf{P}=\mathbf{L}^{T}\mathbf{D}\,\mathbf{L}\ \ (\mathbf{P}=\mathbf{U}^{T}\mathbf{D}\,\mathbf{U})$

Simon, D., 2006, Optimal State Estimation. Kalman, H∞ and Nonlinear Approaches: John Wiley & Sons, Inc., Hoboken, New Jersey.

1D inversion

SVD-like approach

$\mathbf{z}=(\mathrm{Re}\,H_{zi},\mathrm{Im}\,H_{zi}),$
$i=1,\dots,4$

$\mathbf{x}=(\ln\rho)$

measured parameters:

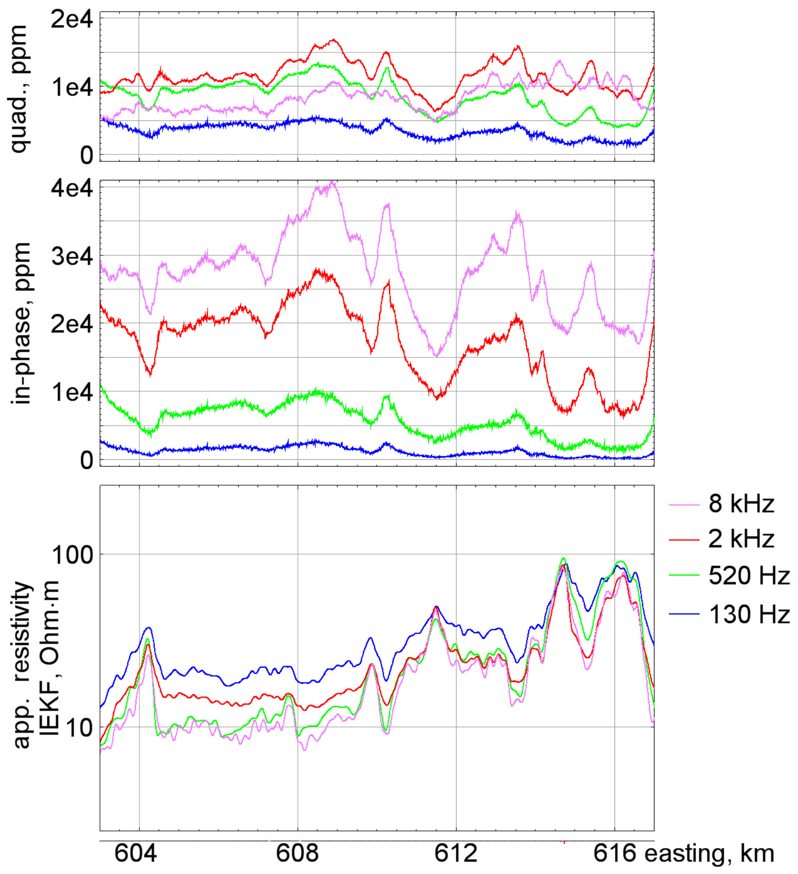
$z-h,\ \ r,\ \ M$

estimated parameters:

$\mathbf{R}=\mathrm{diag}\{\sigma^2_{\mathrm{Re}\,i},\sigma^2_{\mathrm{Im}\,i}\}$
$i=1,\dots,4$

no prognosis step

EM4H: FD AEM



Vovenko, T., Moilanen, E., Volkovitsky, A. and Karshakov, E., 2013, New abilities of quadrature EM systems: Papers of the 13th SAGA Biennial @ 6th International AEM Conference AEM-2013. Mpumalanga, South Africa, 1-4.

1D inversion

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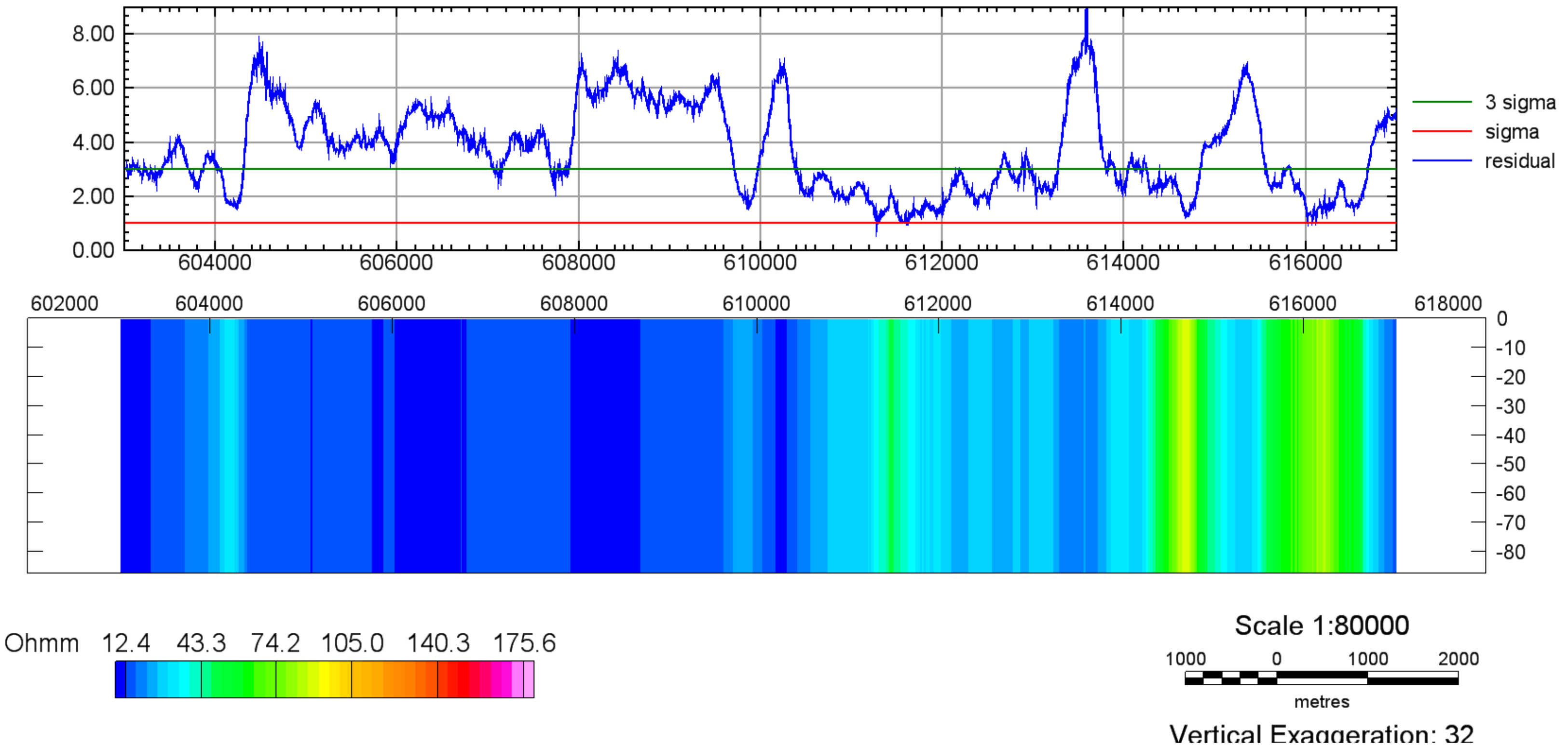
$i=1,\ldots,4$

no prognosis step



1D inversion

EM-4H: FD AEM

SVD-like approach

$\mathbf{z}=(\mathrm{Re}\,H_{zi}, \mathrm{Im}\,H_{zi}),$
$i=1,\dots,4$

$\mathbf{x}=(\ln\rho_1, \ln\rho_2, \ln h_1)$

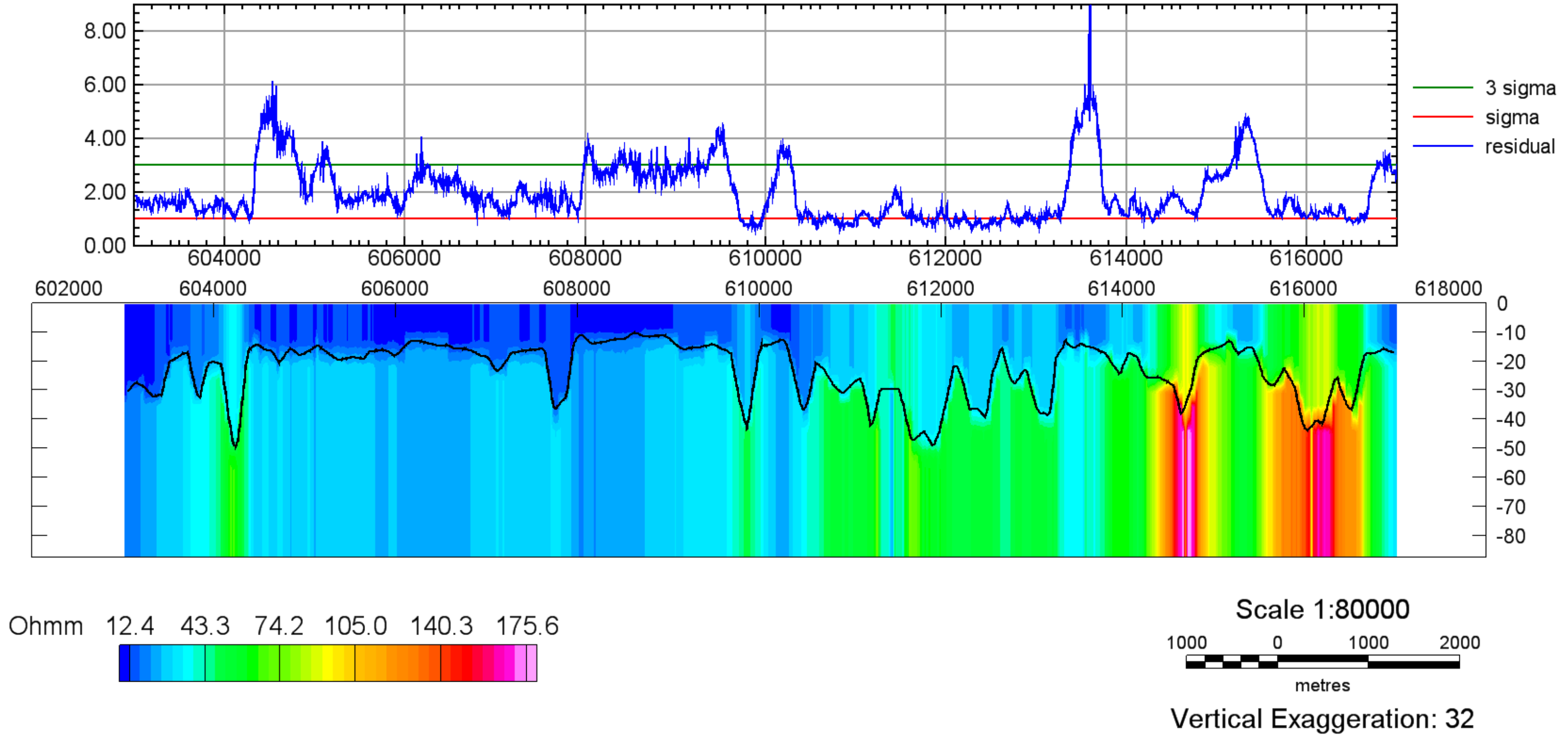
measured parameters:

$z-h,\ r,\ M$

estimated parameters:

$\mathbf{R}=\mathrm{diag}\{\sigma^2_{\mathrm{Re}\,i}, \sigma^2_{\mathrm{Im}\,i}\}$
$i=1,\dots,4$

no prognosis step



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1D inversion

EM-4H: FD AEM

SVD-like approach

$$\mathbf{z} = (\operatorname{Re} H_{zi}, \operatorname{Im} H_{zi}),$$
$$i = 1, \ldots, 4$$

$$\mathbf{x} = (\ln \rho_1, \ln \rho_2, \ln \rho_3, \ln h_1, \ln h_2)$$

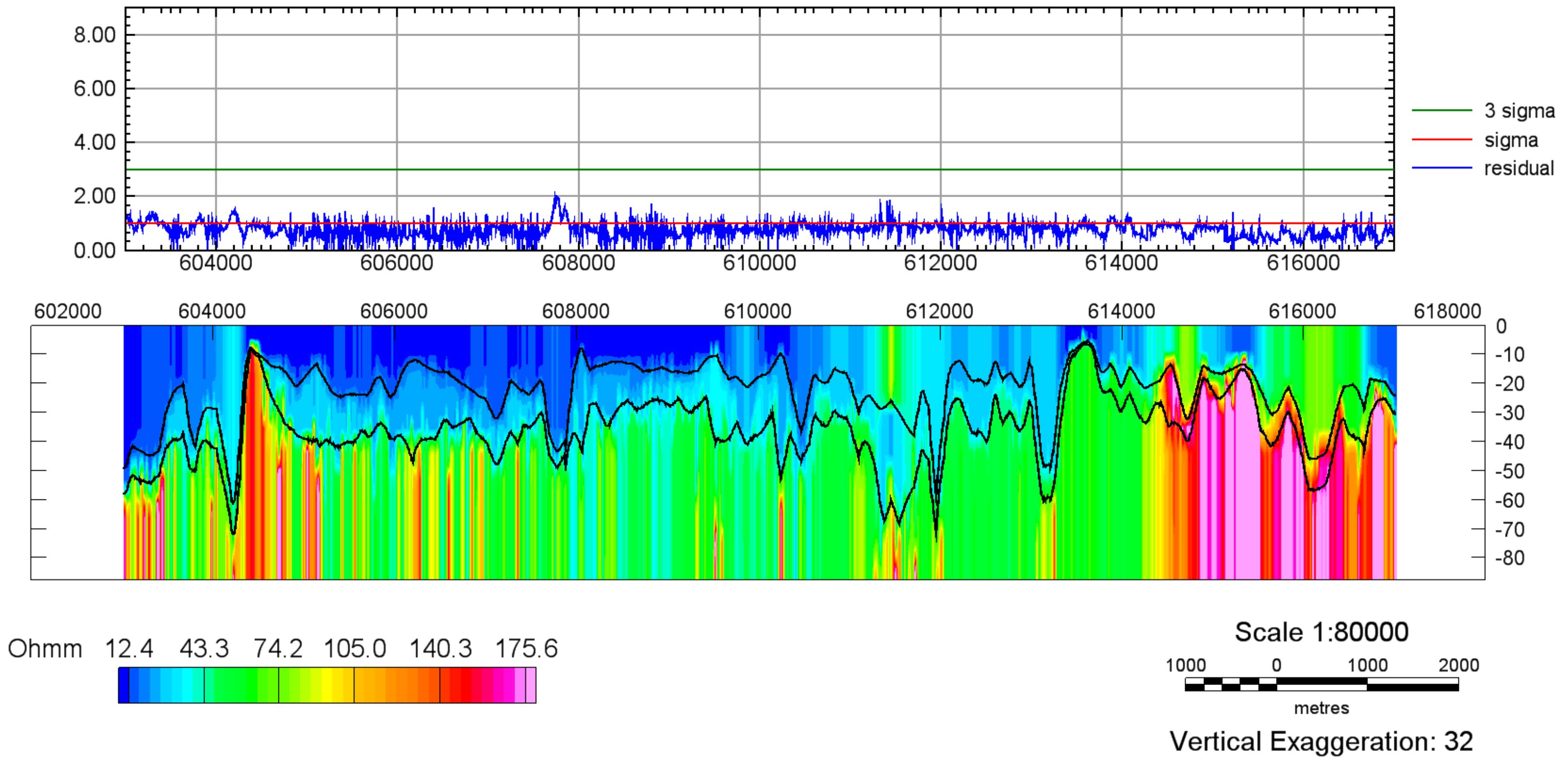
measured parameters:

$z-h, \ r, \ M$

estimated parameters:

$$\mathbf{R} = \operatorname{diag}\{\sigma^2_{\mathrm{Re}\,i}, \sigma^2_{\mathrm{Im}\,i}\}$$
$$i = 1, \ldots, 4$$

no prognosis step



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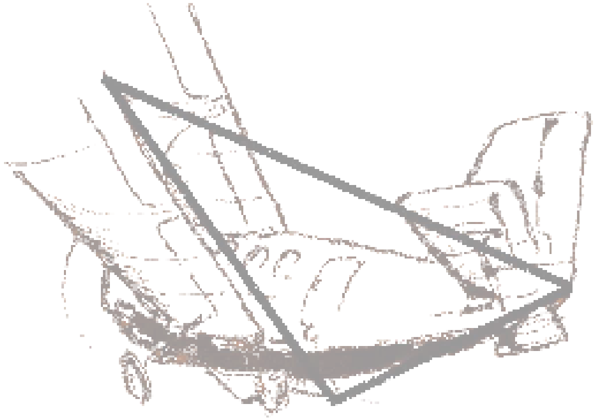
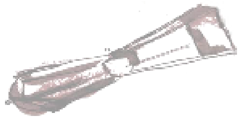
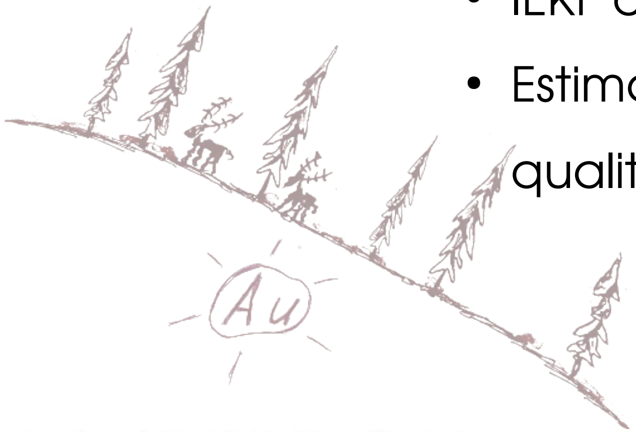


Conclusions

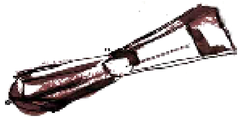
- IEKF was successfully applied to 1D inversion
- IEKF can be considered as a generalized Gauss-Newton method with probabilistic approach
- IEKF can be applied to more complicated inverion problems (2D, 3D...)
- Estimation error covariance matrix allows evaluation of the solution quality: parameter variance, stochastic estimation measure, ...





Pictures: A.K. Volkovitsky

Thank you!



Pictures: A.K. Volkovitsky